Arguments

Lecture 4
Section 2.3

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Outline

Statement Forms

- A statement is a sentence that is true or false.

  "If HSC is in North Carolina, then UVA is in Virginia or pigs can fly."

- A statement form is the logical form of a statement, represented symbolically.

  $p \rightarrow q \vee r$

Arguments

- An argument is a sequence of statements.
- The last statement is the conclusion.
- All the other statements are the premises.
- A mathematical proof is an argument.

Argument Forms

- An argument form is a sequence of statement forms.
- The last statement *form* is the conclusion.
- All the other statement *forms* are the premises.
- A mathematical proof follows an argument form.

Outline

Validity of an Argument Form

- An argument form is valid if its conclusion is true whenever its premises are all true.
- Otherwise, the argument form is invalid.
- An invalid argument form is called a fallacy.

Validity of an Argument

- An argument is valid if its argument *form* is valid, whether or not its premises are true.

Validity of an Argument

- Let the premises be $P_1, P_2, \ldots, P_n$.
- Let the conclusion be C.
- The argument form is valid if

$$P_1 \wedge P_2 \wedge \ldots \wedge P_n \rightarrow C$$

is a tautology.

Example

- I will go fishing today.
- If the boss is in and I go fishing, then I will get fired.
- The boss is in.
- Therefore, I will get fired.

Example

- p = “I will go fishing today.”
- q = “The boss is in.”
- r = “I will get fired.”
- Argument form:

$$\begin{array}{l} p \\ q \wedge p \rightarrow r \\ q \\ \therefore r \end{array}$$

Example

P_1	P_2	P_3	C	$P_1 \wedge P_2 \wedge P_3 \rightarrow C$
p	$q \wedge p \rightarrow r$	q	r	

Example

P_1	P_2	P_3	C	$P_1 \wedge P_2 \wedge P_3 \rightarrow C$
p	$q \wedge p \rightarrow r$	q	r	
T		T	T	
T		T	F	
T		F	T	
T		F	F	
F		T	T	
F		T	F	
F		F	T	
F		F	F	

Example

P_1	P_2	P_3	C	$P_1 \wedge P_2 \wedge P_3 \rightarrow C$
p	$q \wedge p \rightarrow r$	q	r	
T	T	T	T	
T	F	T	F	
T	T	F	T	
T	T	F	F	
F	T	T	T	
F	T	T	F	
F	T	F	T	
F	T	F	F	

Example

P_1	P_2	P_3	C	$P_1 \wedge P_2 \wedge P_3 \rightarrow C$
p	$q \wedge p \rightarrow r$	q	r	
T	T	T	T	T
T	F	T	F	T
T	T	F	T	T
T	T	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T

Outline

Invalid Argument Forms with True Conclusion

- An argument form may be *invalid* even though its conclusion is *true*.
  - If I go fishing, the boss will fire me.
  - The boss fired me.
  - Therefore, I went fishing.
- A true conclusion does not ensure that the argument form is valid, even if all the premises are true.

Invalid Argument Forms with True Conclusion

- Another invalid form with a true conclusion.
  - If $1 + 1 = 2$, then pigs can fly.
  - Pigs can fly.
  - Therefore, $1 + 1 = 2$.

Valid Argument Forms with False Conclusion

- An argument form may be *valid* even though its conclusion is *false*.
  - If I wait until the last minute to do my homework, then it will be a lot easier.
  - I wait until the last minute to do my homework.
  - Therefore, it will be a lot easier.
- A false conclusion does not mean that the argument form is invalid.

Valid Argument Forms with False Conclusion

- Another valid form with a false conclusion.
  - If $1 + 1 = 2$, then pigs can fly.
  - $1 + 1 = 2$.
  - Therefore, pigs can fly.

Outline

Modus Ponens

- Modus ponens is the argument form

$$\begin{array}{r} p \to q \\ p \\ \therefore q \end{array}$$

- This is also called a direct argument.

Examples of Modus Ponens

- If UVA is in Virginia, then HSC is in North Carolina. UVA is in Virginia. Therefore, HSC is in North Carolina.
- If pigs can fly, then UVA is in Virginia. Pigs can fly. Therefore, UVA is in Virginia.

Modus Tollens

- Modus tollens is the argument form

$$\begin{aligned} & p \to q \\ & \sim q \\ \therefore\ & \sim p \end{aligned}$$

- This is also called an indirect argument.
- It is equivalent to replacing $p \to q$ with $\sim q \to \sim p$ and then using modus ponens.

Examples of Modus Tollens

- If UVA is in Virginia, then HSC is in North Carolina. HSC is not in North Carolina. Therefore, UVA is not in Virginia.
- If pigs can fly, then pigs have wings. Pigs do not have wings. Therefore, pigs cannot fly.

Other Argument Forms

- From the specific to the general

$$\begin{array}{l} p \\ \therefore p \vee q \end{array}$$

- From the general to the specific

$$\begin{array}{l} p \wedge q \\ \therefore p \end{array}$$

Other Argument Forms

- Elimination

$$\begin{array}{r} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

- Transitivity

$$\begin{array}{r} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Other Argument Forms

- Division into Cases

$$\begin{array}{l} p \vee q \\ p \rightarrow r \\ q \rightarrow r \\ \therefore r \end{array}$$

Outline

Fallacies

- A fallacy is an invalid argument form.
- Two common fallacies
  - The fallacy of the converse.
  - The fallacy of the inverse.

The Fallacy of the Converse

- The fallacy of the converse is the invalid argument form

$$\begin{array}{r} p \to q \\ q \\ \therefore p \end{array}$$

- This is also called the fallacy of affirming the consequent.

Example

- An example of the fallacy of the converse.

  If pigs can fly, then pigs have wings. Pigs have wings. Therefore, pigs can fly.

Fallacy of the Inverse

- The fallacy of the inverse is the invalid argument form

$$\begin{aligned} p &\to q \\ \sim p & \\ \therefore \sim q & \end{aligned}$$

- This is also called the fallacy of denying the antecedent.

Example

- An example of the fallacy of the inverse.

  If pigs can fly, then pigs have wings. Pigs cannot fly. Therefore, pigs do not have wings.

Outline

Assignment

Collected

- Sec. 2.1: 9, 15, 28, 42.
- Sec. 2.2: 13b, 17.

Assignment

- Read Section 2.3, pages 51 - 61.
- Exercises 1, 3, 6, 7, 11, 22, 23, 24, 29, 31, 40, 42, page 61.